

Equivalent Output Resistance

Consider this simple circuit:

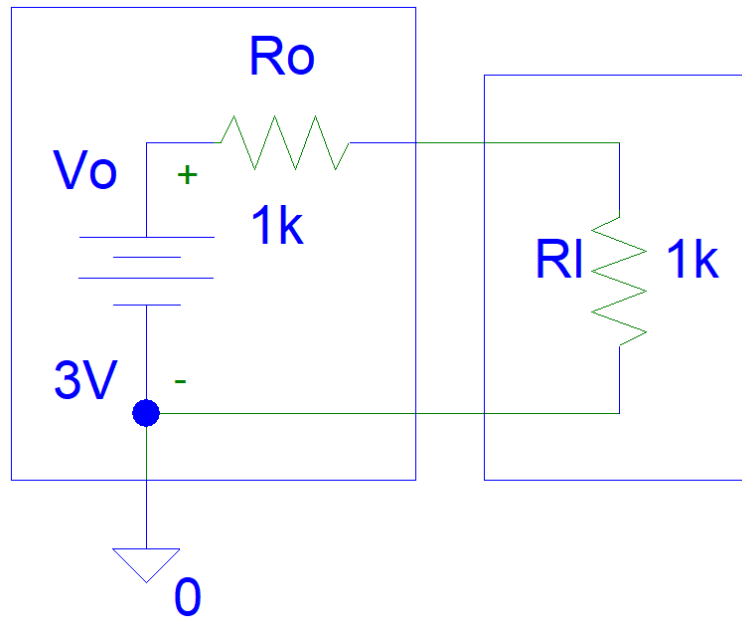


Figure 1: System Circuit

Convert to equivalent resistance:

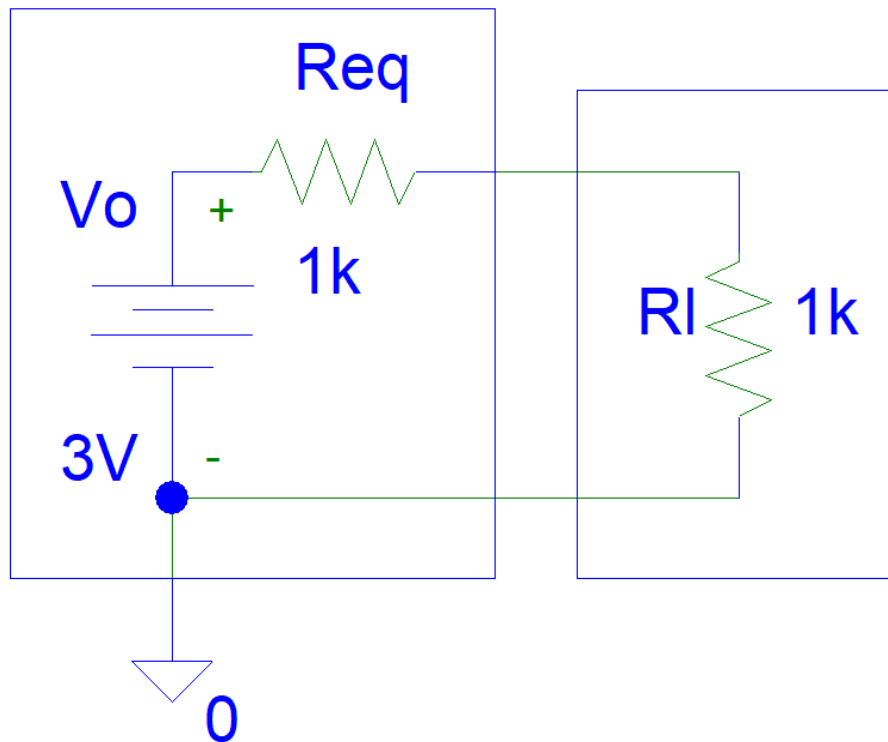


Figure 2: Converting to equivalent resistance

$$\begin{aligned} R_{eq} &= (V_o - V_l) / I_l \\ &= (V_o - V_l) / (V_o / (R_l + R_o)) \\ &= (V_o - V_o * R_l / (R_l + R_o)) / (V_o / (R_l + R_o)) \\ &= (1 - R_l / (R_l + R_o)) * (R_l + R_o) \\ &= (R_l + R_o) - R_l \\ &= R_o \end{aligned}$$

Suppose we add an additional current source:

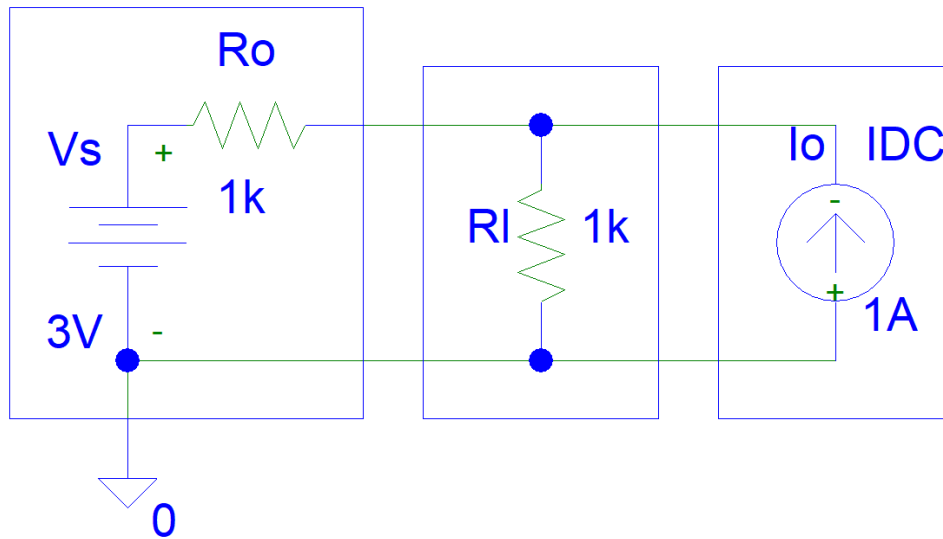


Figure 3: System with extra current source

Convert to equivalent resistance:

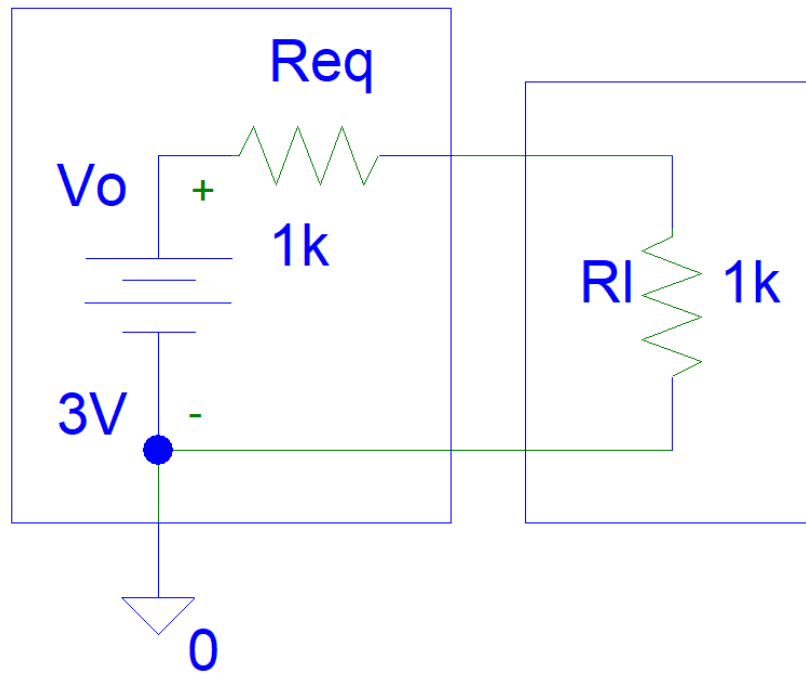


Figure 4: Converting to equivalent resistance

Using superposition theorem:

$$\begin{aligned}
 R_{eq} &= (V_o - V_I) / I_I \\
 &= (V_o - V_I) / (V_o / (R_I + R_o) + I_o) \\
 &= (V_o - V_o * R_I / (R_I + R_o) - I_o * R_I) / (V_o / (R_I + R_o) + I_o) \\
 &= (1 - R_I / (R_I + R_o) - I_o * R_I / V_o) / (1 / (R_I + R_o) + I_o / V_o)
 \end{aligned}$$

Increasing the I_o value will reduce the equivalent resistance.

Suppose we replace the current source with a voltage source:

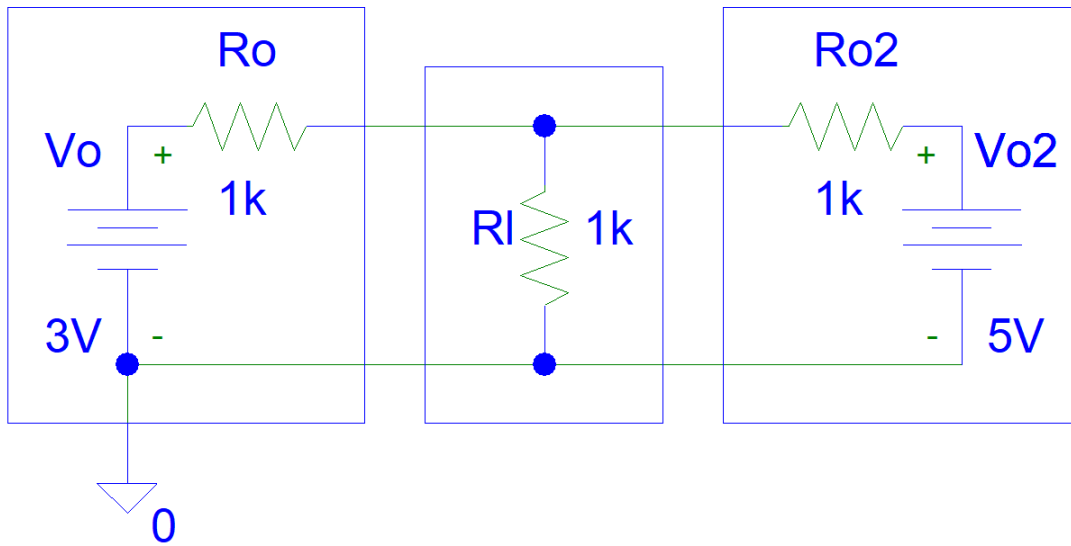


Figure 5: System with extra voltage source

Convert to equivalent resistance:

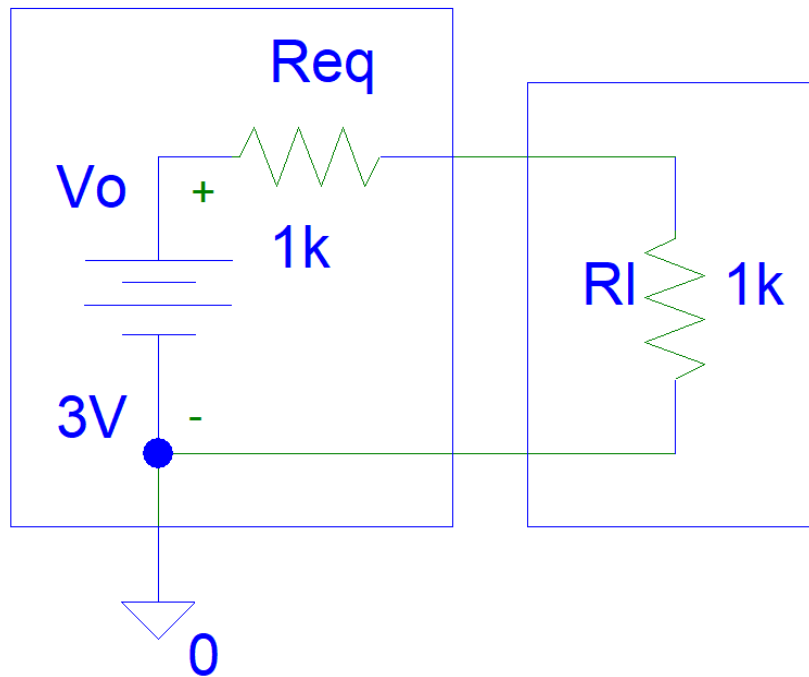


Figure 6: Converting to equivalent resistance

Using superposition theorem:

$$\begin{aligned}
 R_{eq} &= (V_o - V_I) / I \\
 &= (V_o - V_I) / \\
 &= (V_o * (R_I || R_{o2}) / (R_I || R_{o2} + R_o) / R_I + \\
 &+ V_{o2} * (R_I || R_o) / (R_I || R_o + R_{o2}) / R_I) \\
 &= (V_o - V_o * (R_I || R_{o2}) / (R_I || R_{o2} + R_o) \\
 &- V_{o2} * (R_I || R_o) / (R_I || R_o + R_{o2})) / \\
 &= (V_o * (R_I || R_{o2}) / (R_I || R_{o2} + R_o) / R_I + \\
 &+ V_{o2} * (R_I || R_o) / (R_I || R_o + R_{o2}) / R_I)
 \end{aligned}$$

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